# Department of Applied Physics
# Entrance Examination Booklet

## Physics I

*(Answer the 2 Problems in this Booklet)*

August 28 (Tuesday) 9:30 – 11:30, 2018

## REMARKS

1. Do not open this booklet before the start is announced.
2. Inform the staff when you find misprints in the booklet.
3. Answer the two problems in this booklet.
4. Use one answer sheet for each problem (two answer sheets are given). You may use the back side of each answer sheet if necessary.
5. Write down the number of the problem which you answer in the given space at the top of the corresponding answer sheet.
6. You may use the blank sheet of this booklet to make notes, but you must not detach them.
7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.
8. Do not take this booklet and the answer sheets with you after the examination.

| Examinee number | No. |
| --- | --- |

Write down your examinee number above

# Problem 1

[1] Consider a situation where a mass point is thrown at an initial velocity $v$ in the horizontal direction from a position above the surface of the earth. It is assumed that the earth is a uniform sphere of the mass $M$ and the radius $R$. Here, the mass of the mass point is $m$, which is sufficiently smaller than the mass of the earth ($m \ll M$). The gravitational constant is $G$. Also, it is assumed that the height from the ground surface from which the mass point is thrown is sufficiently small and negligible as compared to the radius of the earth. Ignore the effect of air resistance and also the rotation of the earth. The earth can be regarded as the inertia system.

[1.1] Find the value $v_1$ of the initial velocity $v$ of the mass point as long as it does not fall towards the ground surface and keeps orbiting circularly around the earth. Here, use $G$, $M$, $R$ to express $v_1$.

[1.2] Find the value $v_2$ of the minimum initial velocity $v$ of the mass point when it escapes from the gravity of the earth, that is, when it does not draw a closed orbit. Here, use $G$, $M$, $R$ to express $v_2$.

[1.3] When the velocity $v$ of the mass point is $v_1 < v < v_2$, find the length of the long axis of the ellipse $L$ drawn by the mass point. Here, use $G$, $M$, $R$, $v$ to express $L$.

You can use the equation of the trajectory drawn by the mass point:

$$r = \frac{l}{1 + \epsilon \cos(\theta - \theta_0)},$$

which is expressed in the two-dimensional polar coordinate system $(r, \theta)$ in the orbital plane with the center of the earth as the origin. Here, $l = \frac{h^2}{GM}$, $\epsilon = \sqrt{1 + \frac{2h^2E}{G^2M^2m}}$, $h$ is double of the area velocity ($h = r^2\frac{\mathrm{d}\theta}{\mathrm{d}t}$), $E$ is the total energy of the mass point (given that its potential energy is 0 at infinitely far distances), $\theta_0$ is a constant depending on the setting of the coordinate axis.

[2] Consider a system S, where two mass points of the same mass $m$ are connected by a massless rod of length $l$, as shown in Figure 1. The center of mass of S circularly orbits with a constant angular velocity $\omega_0$ around the center of the earth. We set the moving frame as shown in Figure 1; that is, the origin of the moving frame is set to the center of mass of S, the $x$-axis is along an imaginary line directing the mass center S from the center of the earth, the $y$-axis is along the direction of the velocity of the center of mass, and the $z$-axis is taken to be right-handed. Here, it is assumed that the mass points move only in the $x$-$y$ plane. Also, a vector connecting the center of mass of S from the center of the earth is defined as $\boldsymbol{R}_0$, and the angle $\phi$ is defined as shown in Figure 1. Answer the following questions by using the gravitational constant as $G$, the mass of the earth as $M$, and $m \ll M$.

[2.1] Find the angular velocity $\omega_0$ of the circular motion of S, by using $G$, $M$, $R_0 \equiv |\boldsymbol{R}_0|$.

[2.2] Find the moment of inertia $I$ of S around the $z$-axis.

[2.3] The moment of the force acting on S around the $z$-axis is expressed as $N_z = \boxed{\phantom{xxxx}}\, l^2\omega_0^2$, when $l$ is sufficiently smaller than $R_0$. Express the coefficient of $l^2\omega_0^2$ denoted by $\boxed{\phantom{xxxx}}$, using $m$ and $\phi$. Here, the moment of force is driven only from the gravity of the earth.

[2.4] For the angle $\phi$, find all the values $\phi_0$ in the range of $0 \leq \phi_0 < 2\pi$ when $N_z = 0$, that is, the equilibrium state of the relative motion. Also, find the angular frequency of the small amplitude oscillation of $\phi$ around a stable equilibrium position $\phi = \phi_0$, by using $\omega_0$.

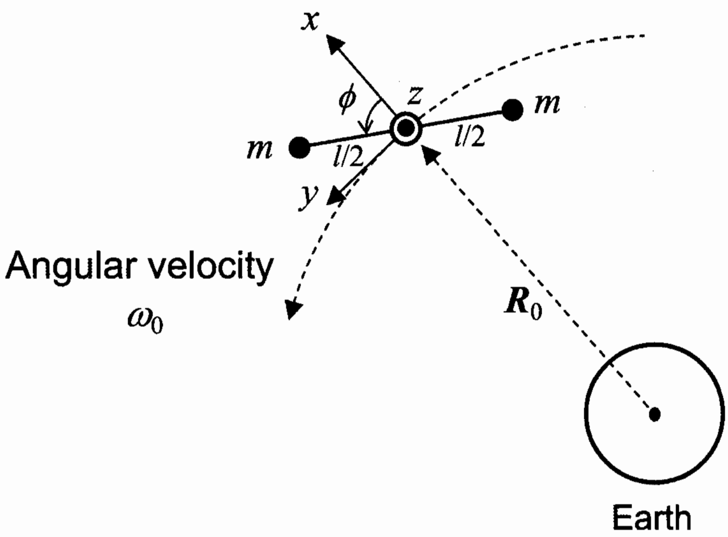


Figure 1

## Problem 2

[1] A spatially uniform magnetic field is applied along the axial direction of a metallic disk with the radius $R$ and height $L$ in the vacuum (Figure 1). The magnetic flux density oscillates as $B(t) = \mu_0 H_0 \sin(2\pi f t)$, assuming it has positive values when the field direction is pointed upward, as shown in Figure 1. Here $\mu_0$ is the vacuum permeability, $t$ is the time and $f$ is the frequency. For simplicity, neglect the skin effect in the metallic disk and assume that the metallic disk is not magnetized. The heat capacity and electrical resistivity, denoted as $C$ and $\rho$, respectively, are independent of temperature and frequency.

[1.1] Consider a cylinder-shaped portion of the metallic disk with the radius $r$ (Figure 2). The thickness, $\mathrm{d}r$, of this cylinder is negligibly small. Express the induced electric field $E$ in the cylinder, resulting from the oscillation of the magnetic field. Here, the positive direction of the electric field is counter-clockwise as viewed from the top of Figure 1.

[1.2] Next, consider the whole metallic disk. It should heat up due to the oscillation of the magnetic field. Find the expression for the electrical power $P$ corresponding to the induction heating.

[1.3] Calculate the temperature increase of the metallic disk with one significant digit for an oscillating magnetic field applied for 1 second. Here, $H_0 = 8 \times 10^4\,\mathrm{A\,m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\,\Omega\,\mathrm{s\,m^{-1}}$, $R = 0.01\,\mathrm{m}$, $L = 0.001\,\mathrm{m}$, $\rho = 4 \times 10^{-8}\,\Omega\,\mathrm{m}$, $C = 0.5\,\mathrm{J\,K^{-1}}$, and $f = 10$ Hz. Assume that the heat leak from the metallic disk is negligible.

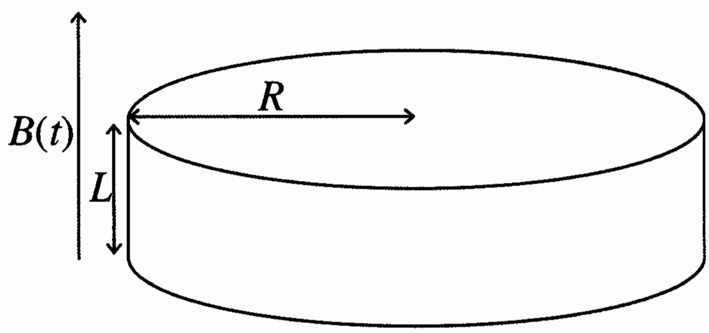


Figure 1

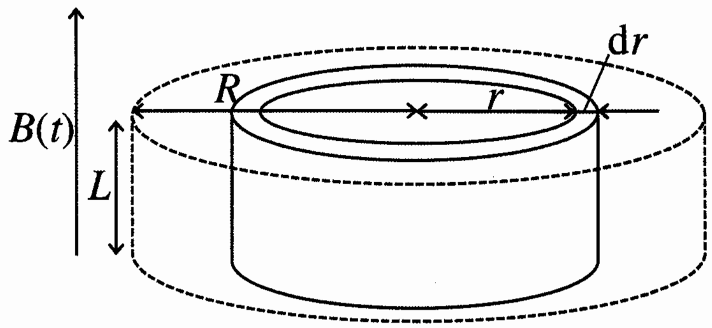


Figure 2

[2] Next, consider the magnetic flux density generated in the vacuum by a circular current loop of the radius $a$ carrying current $I$. The central axis of the loop is along the $x$-axis. The vacuum

permeability is $\mu_0$, and the thickness of the wire is negligibly small. Answer the following questions.

[2.1] Consider the current loop located on the $x = 0$ plane. Express the magnitude of the magnetic flux density at the point Q on $x$-axis as a function of its coordinate $x$ when the current flows as shown in Figure 3.

[2.2] Two parallel current loops are placed on the $x = -b$ and $x = b$ planes, respectively (Figure 4). Assuming that the current directions in these loops are opposite as shown in Figure 4, express the magnitude of the total magnetic flux density at an arbitrary point Q, located on the $x$-axis as a function of its coordinate $x$. Find an approximate form for the magnitude of the magnetic flux density up to the first order terms in $x$ for a given point Q, located in the vicinity of origin O on the $x$-axis. Here $b$ is a positive value.

[2.3] In the configuration shown in Figure 4, the current direction of the right loop is reversed, so that both loops have the same current direction. Find an approximate form for the magnitude of the magnetic flux density up to the second order in $x$ for a given point Q, located in the vicinity of origin O on the $x$-axis. Also, find the relation between $a$ and $b$ when the second-order term becomes 0 in the obtained approximate form.

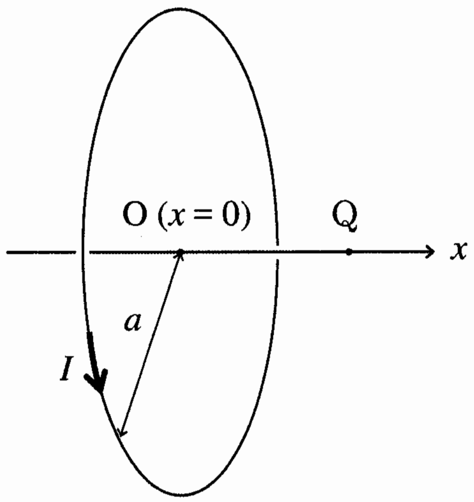


Figure 3

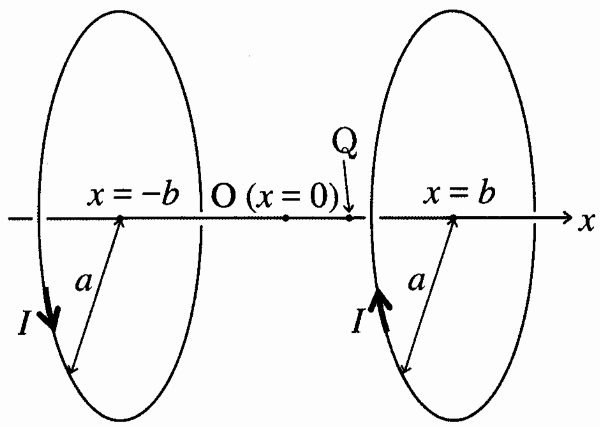


Figure 4

# Department of Applied Physics
# Entrance Examination Booklet

## Physics II

*(Answer 3 Problems among the 4 Problems in this Booklet)*

August 28 (Tuesday) 13:00 – 16:00, 2018

## REMARKS

1. Do not open this booklet before the start is announced.
2. Inform the staff when you find misprints in the booklet.
3. Choose three problems among the four problems in this booklet, and answer the three problems.
4. Use one answer sheet for each problem (three answer sheets are given). You may use the back side of each answer sheet if necessary.
5. Write down the number of the problem which you answer in the given space at the top of the corresponding answer sheet.
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| Examinee number | No. |
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Write down your examinee number above

## Problem 1

Consider one-dimensional quantum mechanical motion of particles of mass $m$, obeying the Schrödinger equation, being scattered by a delta-function potential barrier $U(x) = \alpha\delta(x)$ $(\alpha > 0)$ at the origin $x = 0$. Suppose wave packets are spatially extended such that particles are describable by monochromatic plane waves with the energy $E(> 0)$. We neglect the internal degrees of freedom of particles including spin. $\hbar$ is the normalized Planck constant, i.e., $h$ divided by $2\pi$. Defining the wave number of a particle as $k = \sqrt{2mE}/\hbar$, and the dimensionless variable $C$ as $C = m\alpha/(\hbar^2 k)$, answer the following questions.

[1] Suppose an incident particle with the wave number $k$ reaches the origin from $x < 0$ as shown in Figure 1. The transmission coefficient and the reflection coefficient are defined as $t$ and $r$, respectively.

[1.1] Express boundary conditions of the wave function $\psi(x)$ at $x = 0$. Note that the boundary condition for the spatial derivative of the wave function is obtained by integrating the Schrödinger equation $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$ over a narrow range in the vicinity of $x = 0$.

[1.2] Express the transmission coefficient $t$ and the reflection coefficient $r$ as functions of $C$.

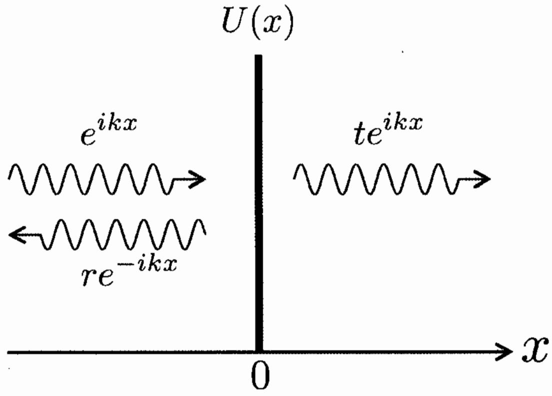


Figure 1

[2] Consider the interference between two identical non-interacting particles with the energy $E$ incident at the origin from the opposite directions. Note that the one-particle state changes by the scattering as

$$\begin{aligned} \psi_+(x) &\rightarrow t\psi_+(x) + r\psi_-(x), \\ \psi_-(x) &\rightarrow r\psi_+(x) + t\psi_-(x), \end{aligned}$$

where $\psi_+(x)$ and $\psi_-(x)$ are the right-going and left-going waves, and $t$ and $r$ are those obtained in [1.2].

[2.1] Show that when the two-particle wave function is anti-symmetric under the exchange of position coordinates, the two particles are scattered into the opposite directions with probability 1, irrespective of the choice of $\alpha$ and $E$.

[2.2] Show that when the two-particle wave function is symmetric under the exchange of position coordinates, the two particles are scattered into the opposite directions for $\alpha \to 0$ or $\alpha \to \infty$, but into the same directions with probability 1 for $\alpha = \alpha_0$. Obtain $\alpha_0$ as a function of $E$.

Next, we add a potential barrier $U_L(x) = \alpha\delta(x - L)$ at $x = L(> 0)$ as shown in Figure 2.

[3] The transmission probability of an incident particle from $x < 0$ to $x > L$ becomes 1 while its reflection probability to $x < 0$ becomes 0. Express $L$ as a function of $k$ and $C$.

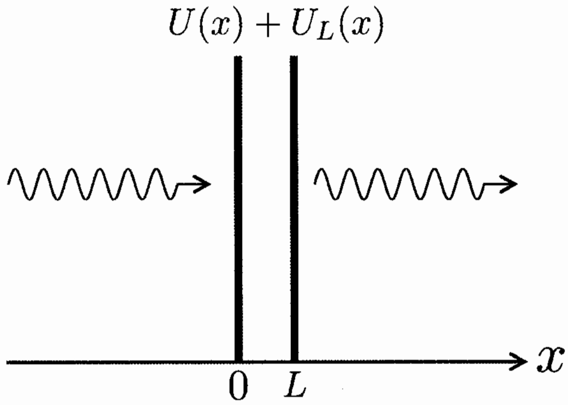


Figure 2

## Problem 2

Consider the thermal equilibrium state of a system consisting of $N$ classical particles of the same kind in a container with volume $V$ at temperature $T$. Behavior of this system is well described by the van der Waals equation of state,

$$P = \frac{nk_{\mathrm{B}}T}{1-bn} - an^2, \tag{1}$$

where $n = N/V$ and $k_{\mathrm{B}}$ denote particle number density and the Boltzmann constant, respectively, and $a$ and $b$ are positive constants.

[1] The equation of state (1) has a point at a certain temperature $T = T_c$ and a certain particle number density $n = n_c$ where

$$\left(\frac{\partial P}{\partial n}\right)_T = 0 \quad \text{and} \quad \left(\frac{\partial^2 P}{\partial n^2}\right)_T = 0. \tag{2}$$

Find $T_c$ and $n_c$, and the corresponding pressure $P_c$ at this point.

[2] In the case that $T > T_c$ and $n = n_c$, derive isothermal compressibility $K_T$,

$$K_T = -\frac{1}{V} \cdot \left(\frac{\partial V}{\partial P}\right)_T, \tag{3}$$

by using $k_{\mathrm{B}}$, $n_c$, $T$ and $T_c$.

In the following, the van der Waals equation of state (1) will be derived. Coordinates and momentum of particle $i$ $(i = 1, 2, \cdots, N)$ are denoted by $\boldsymbol{r}_i = (r_{ix}, r_{iy}, r_{iz})$ and $\boldsymbol{p}_i = (p_{ix}, p_{iy}, p_{iz})$, respectively. Mass of each particle is denoted by $m$. The particles interact via a two-body central-force potential function, $u(r)$, where $r$ denotes the inter-particle distance. This $u(r)$ is described as,

$$u(r) = \begin{cases} +\infty & (r < \ell) \\ -\varepsilon \cdot (\ell/r)^6 & (r \geq \ell) \end{cases} \tag{4}$$

with $\ell$ and $\varepsilon$ being positive constants. Energy $E$ of this system is accordingly expressed as

$$E = \sum_{i=1}^{N} \frac{\boldsymbol{p}_i^2}{2m} + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} u(|\boldsymbol{r}_i - \boldsymbol{r}_j|). \tag{5}$$

Let us define

$$Z_0(T,V,N) = \frac{1}{N!}\frac{V^N}{h^{3N}} \prod_{i=1}^{N} \left[\int_{-\infty}^{\infty} \mathrm{d}p_{ix} \int_{-\infty}^{\infty} \mathrm{d}p_{iy} \int_{-\infty}^{\infty} \mathrm{d}p_{iz} \exp\left(-\frac{1}{k_{\mathrm{B}}T} \cdot \frac{\boldsymbol{p}_i^2}{2m}\right)\right] \tag{6}$$

and

$$A(T,V,N) = \frac{1}{V^N} \int_V \mathrm{d}\boldsymbol{r}_1 \int_V \mathrm{d}\boldsymbol{r}_2 \cdots \int_V \mathrm{d}\boldsymbol{r}_N \exp\left[-\frac{1}{k_{\mathrm{B}}T} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} u(|\boldsymbol{r}_i - \boldsymbol{r}_j|)\right], \tag{7}$$

where $h$ denotes the Planck constant, and space integrations, $\int_V \mathrm{d}\boldsymbol{r}_i$, are taken over the volume, $V$, of the container. The partition function of this system, $Z(T,V,N)$, is then expressed as $Z(T,V,N) = Z_0(T,V,N) \cdot A(T,V,N)$.

The function, $Z_0(T, V, N)$, is the partition function of the ideal gas, and $F_0(T, V, N) = -k_B T \log Z_0(T, V, N)$ is the Helmholtz free energy of the ideal gas. Hereafter, $N$ is assumed to be large enough to approximate $\log(N!) \approx N(\log N - 1)$. The following equality

$$\int_{-\infty}^{\infty} e^{-x^2} \mathrm{d}x = \sqrt{\pi} \tag{8}$$

may also be used, if necessary.

[3] Derive an expression for $F_0(T, V, N)$ using $T$, $V$, $N$, $m$, $k_B$ and $h$. Using this $F_0$, derive pressure $P_0$, entropy $S_0$, chemical potential $\mu_0$ and internal energy $U_0$ for the ideal gas.

In the following, the Helmholtz free energy of this system is approximated in high-temperature ($\varepsilon \ll k_B T$) and low-density limits.

The particles cannot take configurations where the interaction potential (4) approaches to infinity. Therefore there is a region in the vicinity of each particle where no other particle can exist. The volume of this region per particle is called the "excluded volume" and denoted as $v$. Here we consider low-density limit. Accordingly, the effect of three or more particles is neglected and only the two-particle effect is taken into account, which results in $v = 2\pi\ell^3/3$. At high temperature ($\varepsilon \ll k_B T$) and low density limits, $A$ in equation (7) is approximated as

$$A \approx \left(\frac{V - Nv}{V}\right)^N \left[1 - \frac{4\pi}{V}\int_{\ell}^{\infty} r^2 \frac{u(r)}{k_B T}\mathrm{d}r\right]^{N(N-1)/2}. \tag{9}$$

Answer the following quations using this expression.

[4] Derive the Helmholtz free energy $F(T, V, N)$, using $T$, $V$, $N$, $\varepsilon$, $m$, $v$, $k_B$ and $h$.

[5] Derive pressure $P$ of this system, using $T$, $n$, $\varepsilon$, $v$ and $k_B$.

[6] Derive constants $a$ and $b$ of the van der Waals equation of state (1), using $\varepsilon$ and $v$.

## Problem 3

Consider the propagation of an electromagnetic plane wave with the angular frequency $\omega$ in isotropic media. The electromagnetic wave passes through from an air space with the refractive index $n_0$ towards a sufficiently thick glass with the refractive index $n_g$. The media are separated by planar interfaces parallel to the $xy$-plane, and the incident wave propagates perpendicular to the interfaces. You can use the Maxwell's equation $\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$, where $\boldsymbol{E}$ and $\boldsymbol{H}$ denote the electric and magnetic fields, respectively, and $\mu$ is the magnetic permeability of the medium. For simplicity, the magnetic permeability of all the media $\mu$ are assumed to be equal to the permeability of the vacuum $\mu_0$. All the media are uniform and isotropic. The electromagnetic wave is not absorbed in the media. Below, $c$ denotes the speed of the light in the vacuum, and the complex notation is used to describe the electromagnetic waves. Answer the following questions.

[1] Consider the case where the electromagnetic wave propagates from the air to the glass (Figure 1). The position of the air/glass interface is $z = 0$. Assume that the incident electric field is along the $y$-direction, and is described as $E^{(\mathrm{i})}(z,t) = E_0^{(\mathrm{i})} \exp\left[-i\left(\frac{\omega n_0}{c} z + \omega t\right)\right]$. Accordingly, the incident magnetic field is along the $x$-direction, and is written as $H^{(\mathrm{i})}(z,t) = H_0^{(\mathrm{i})} \exp\left[-i\left(\frac{\omega n_0}{c} z + \omega t\right)\right]$.

[1.1] Derive the expression for $H_0^{(\mathrm{i})}$ using $E_0^{(\mathrm{i})}$.

[1.2] Derive the boundary conditions for the electric and magnetic fields in the directions parallel to the $z = 0$ plane. The effect of the surface current is negligible. The amplitudes of the incident, reflected and transmitted electric fields along the $y$-direction are given by $E_0^{(\mathrm{i})}$, $E_0^{(\mathrm{r})} \equiv E^{(\mathrm{r})}(z=0, t=0)$ and $E_0^{(\mathrm{t})} \equiv E^{(\mathrm{t})}(z=0, t=0)$, and the amplitudes of the incident, reflected and transmitted magnetic fields along the $x$-direction are given by $H_0^{(\mathrm{i})}$, $H_0^{(\mathrm{r})} \equiv H^{(\mathrm{r})}(z=0, t=0)$ and $H_0^{(\mathrm{t})} \equiv H^{(\mathrm{t})}(z=0, t=0)$, respectively.

[1.3] The amplitude reflection coefficient is written as $r_0 = E_0^{(\mathrm{r})}/E_0^{(\mathrm{i})}$. Derive the amplitude reflection coefficient using $n_0$ and $n_g$.

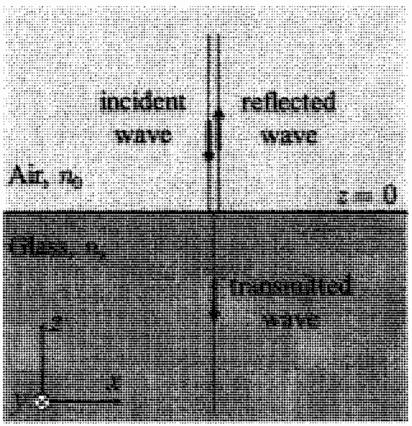


Figure 1

[2] Next, consider a dielectric multilayer film stacked on top of the glass as depicted in Figure 2. The multilayer film is composed of $2N$ stacking layers. The corresponding interfaces are flat and are numbered as $0, 1, 2, \cdots, l-1, l, \cdots, 2N-1, 2N$, as seen from the air side to the

glass side. The position of the $l$-th interface is given by $z = z_l$, and the region between the interfaces $l-1$ and $l$ is called the $l$-th layer, having a refractive index $n_l$. Here the air space is denoted as the zero-th layer. Consider the case where the electromagnetic wave propagates perpendicularly to the interfaces, and the electric and magnetic fields in the $l$-th layer are along the $y$- and $x$-direction, respectively, which are given by

$$
\begin{aligned}
E_l(z,t) &= \left\{E_l^{(-)} \exp\left[-i\frac{\omega n_l}{c}(z-z_l)\right] + E_l^{(+)} \exp\left[i\frac{\omega n_l}{c}(z-z_l)\right]\right\} \exp(-i\omega t), \\
H_l(z,t) &= \left\{H_l^{(-)} \exp\left[-i\frac{\omega n_l}{c}(z-z_l)\right] + H_l^{(+)} \exp\left[i\frac{\omega n_l}{c}(z-z_l)\right]\right\} \exp(-i\omega t).
\end{aligned}
$$

[2.1] Derive the expressions for the amplitudes $H_l^{(-)}$ and $H_l^{(+)}$ using $E_l^{(-)}$ and $E_l^{(+)}$, respectively.

[2.2] Derive the following recursion relation valid for $l = 1, 2, \cdots, 2N$, and the expression for the coefficient $\alpha_l$ by taking into account the boundary conditions at the $(l-1)$-th interface. Here, we describe the thickness of the $l$-th layer as $d_l$ and define $\Delta_l = \frac{n_l \omega d_l}{c}$.

$$
\frac{E_{l-1}^{(-)} - E_{l-1}^{(+)}}{E_{l-1}^{(-)} + E_{l-1}^{(+)}} = \alpha_l \frac{E_l^{(-)} e^{-i\Delta_l} - E_l^{(+)} e^{i\Delta_l}}{E_l^{(-)} e^{-i\Delta_l} + E_l^{(+)} e^{i\Delta_l}}
$$

[2.3] The flat layer L has the refractive index $n_{\mathrm{L}}$, and the flat layer H the refractive index $n_{\mathrm{H}}$. They are alternatively stacked as LHLHL$\cdots$HLH ($2N$ layers) on the glass from the air side to the glass side. The thickness of $l$-th layer satisfies $\Delta_l = \frac{\pi}{2}$, and the multilayer film is utilized as an anti-reflection layer. Derive the expression for the amplitude reflection coefficient $r_1 = E_0^{(+)}/E_0^{(-)}$ and for the perfect anti-reflection situation using $n_0, n_{\mathrm{g}}, n_{\mathrm{L}}$, $n_{\mathrm{H}}$, and $N$.

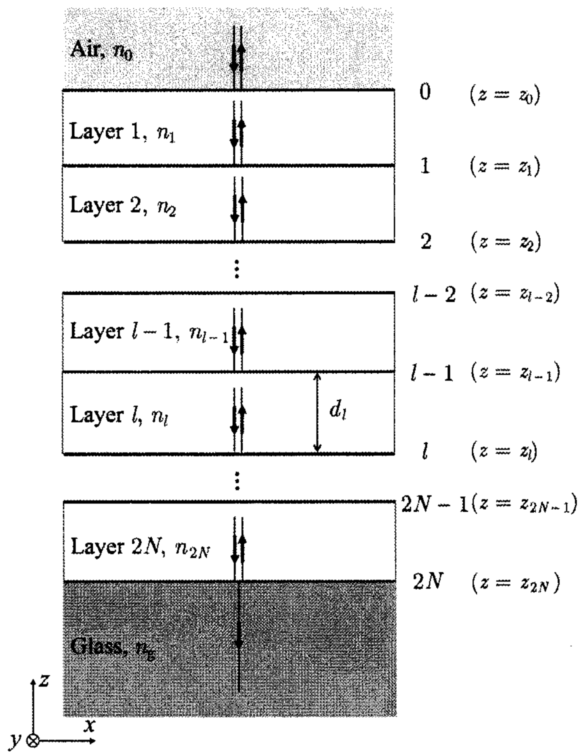
Air, $n_0$
Layer 1, $n_1$
Layer 2, $n_2$
Layer $l-1$, $n_{l-1}$
Layer $l$, $n_l$
$d_l$
Layer $2N$, $n_{2N}$
Glass, $n_g$
0 $(z = z_0)$
1 $(z = z_1)$
2 $(z = z_2)$
$l-2$ $(z = z_{l-2})$
$l-1$ $(z = z_{l-1})$
$l$ $(z = z_l)$
$2N-1$ $(z = z_{2N-1})$
$2N$ $(z = z_{2N})$
z
x
y

Figure 2

## Problem 4

A metallic thick plate M with the area $A$ and the thickness $L$ ($A \gg L^2$) and an n-type semiconductor thick plate S with the same shape as M are faced parallel to each other in the vacuum, as shown in Figure 1. In the conduction band in S, there are electrons excited from the donor impurities. The distance between M and S is $d$, where $A \gg d^2$. As shown in the energy diagram in Figure 2, the chemical potential for an electron in the isolated M measured from the vacuum level is $-W$, where $W > 0$. In the isolated S, the energy of the conduction band bottom is $-\chi_c$, the chemical potential for an electron is $-\phi_s$, and the energy of the valence band top is $-\chi_v$ as measured from the vacuum level ($\chi_v > \phi_s > \chi_c > 0$). The electron charge is denoted by $-e$. The dielectric constant of vacuum is denoted as $\varepsilon_0$. The interiors of M and S are uniform and isotropic. The effects of surface electronic levels or interface electronic levels are not taken into consideration. The density of states at the Fermi level in M, the band gap in S, the values of effective mass at the conduction band bottom and the valence band top in S remain unchanged for any electron number. Electron emission into the vacuum is negligibly small, and deformation of M and S is not considered.

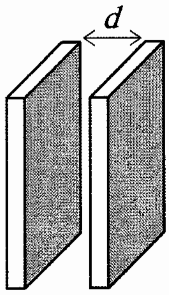


Figure 1

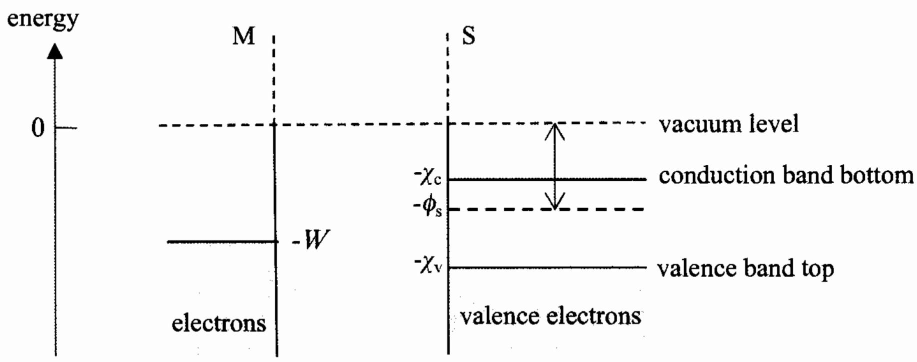


Figure 2

[1] M and S are connected with a conducting wire with a negligible thickness. The charge is accordingly transferred between M and S through the wire, thereby inducing charge surface densities $-\rho$ and $\rho$ on the facing sides of M and S, respectively. In [1], the thickness of the induced charge accumulation layer can be assumed to be zero.

[1.1] Write the thermodynamic expression for the chemical potential, and explain why the charge is transferred.

[1.2] Calculate the magnitude of $\rho$.

[1.3] Write the magnitude of the attracting force between M and S, using $|W - \phi_s|$.

[1.4] Varying the distance between M and S around $d$ as $d + \delta \sin(\Omega t)$ where $t$ is time, an oscillating current with the amplitude $I_0$ is induced between M and S. The resistance and the self inductance of M and S are negligibly small. Write $I_0$ using $|W - \phi_s|$. Here, $\delta \ll d$, and calculate $I_0$ up to the first order of $\delta/d$.

[2] Consider a junction between M and S. For $W > \phi_s > \chi_c$, a region called the depletion layer is formed in the vicinity of the junction between M and S, where the electronic states are spatially modulated as shown in the energy diagram in Figure 3. This is because ion core charge in the layer comes up to the surface. The origin of the $x$ axis is set at the junction between M and S, and its positive direction is along the normal direction from M to S.

[2.1] Write potential barrier seen from S, $\Delta_S$, at the junction.

[2.2] Applying voltage between M and an arbitrary point in S far from the junction, a current flowing across the junction exhibits a rectification behavior (meaning that the magnitude of the current depends on the sign of the voltage). Explain the reason, based on the electronic states distributed around the junction. Here, assume that the energy distribution of electrons in S approximates the Maxwell-Boltzmann distribution function.

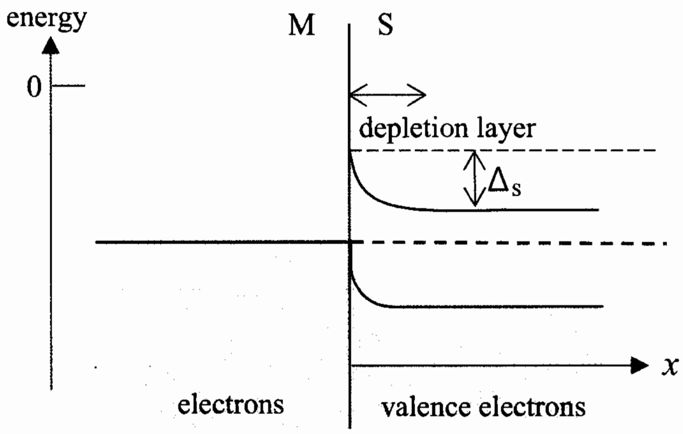


Figure 3