Department of Applied Physics
Entrance Examination Booklet

Physics I

(Answer the 2 Problems in this Booklet)

August 30 (Tuesday) 9:30 – 11:30, 2016

REMARKS

1. Do not open this booklet before the start is announced.
2. Inform the staff when you find misprints in the booklet.
3. Answer the two problems in this booklet.
4. Use one answer sheet for each problem (two answer sheets are given). You may use the back side of each answer sheet if necessary.
5. Write down the number of the problem which you answer in the given space at the top of the corresponding answer sheet.
6. You may use the blank sheet of this booklet to make notes, but you must not detach them.
7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.
8. Do not take this booklet and the answer sheets with you after the examination.

Examinee number	No.

Write down your examinee number above

Problem 1

[1] Three objects A, B, and C of mass m are connected by two identical springs with the spring constant k, and are placed on a horizontal surface with friction, as shown in Fig. 1. The maximum static force and kinetic frictional force are μmg and $\frac{2\mu}{3}mg$, respectively. Here, μ is the maximum static friction coefficient and g is the acceleration of gravity. Initially the springs are at their equilibrium length and these objects are at rest. From this initial state, the object C is forced to move in the positive x direction at a constant speed v_0, as indicated in Fig. 1. These objects move only along the x direction. Answer the following questions [1.1]–[1.4].

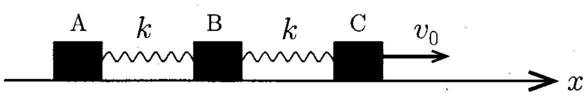

Figure 1

[1.1] Obtain the elapsed time t_0 from when the object C starts to move to when the object B begins to move.

[1.2] For a certain period of time after the object B begins to move, the object A remains at rest. Let $x_B(t)$ be the position of center of gravity of the object B at time t. Derive the equation of motion for the object B. Here, set $t = 0$ for when the object B begins to move and $x_B(t = 0) = 0$.

[1.3] Obtain $x_B(t)$ by solving the equation of motion, derived in [1.2].

[1.4] Assume that t_0 is sufficiently small compared to $\sqrt{\frac{m}{k}}$, which has the dimension of time (i.e. $t_0\sqrt{\frac{k}{m}} \ll 1$). Substituting $t = t_0$ into $x_B(t)$ obtained in [1.3], find an approximate expression for $x_B(t = t_0)$ in terms of t_0, v_0, k, and m, only. Based on the obtained expression, briefly explain whether the object A remains at rest at $t = t_0$ or not.

[2] Three mass points A, B, and C of mass m are connected by two identical springs of the spring constant k_0 and equilibrium length l, as shown in Fig. 2(a). These mass points can only move along the x direction without any friction. In addition to the forces exerted by the springs, there is another force acting on each mass point from the following potential:

$$U(x) = k_1 l^2 \left[-\frac{1}{2}\left(\frac{x}{l}\right)^2 + \frac{1}{4}\left(\frac{x}{l}\right)^4 \right] \quad (k_1 > 0).$$

The schematic of $U(x)$ is shown in Fig. 2(b). These mass points are initially at the equilibrium positions; A, B, and C are at $x = -l$, 0, and l, respectively. Answer the following questions [2.1]–[2.3].

[2.1] Suppose that the value of k_1 is set such that these equilibrium positions are stable. A small deviation of the system from the equilibrium leads to small oscillations of these mass points. The displacements from the equilibrium positions are denoted by q_A, q_B, and q_C for A, B, and C, respectively. Assuming that the displacements from the equilibrium positions are sufficiently small, write down the equations of motion to the first order in terms of q_A, q_B, and q_C. Also, find all the eigenfrequencies for such small oscillations.

[2.2] For k_1 larger than a certain value k_c, one of the normal modes of the small oscillations is unstable. For $k_1 > k_c$, among the eigenfrequencies found in [2.1], which one corresponds to this unstable normal mode? Also, find this k_c.

[2.3] At $k_1 = \frac{4k_c}{9} (< k_c)$, obtain the ratio of the displacements for the mass points A, B, and C (i.e. $q_A : q_B : q_C$) for the normal mode considered in [2.2].

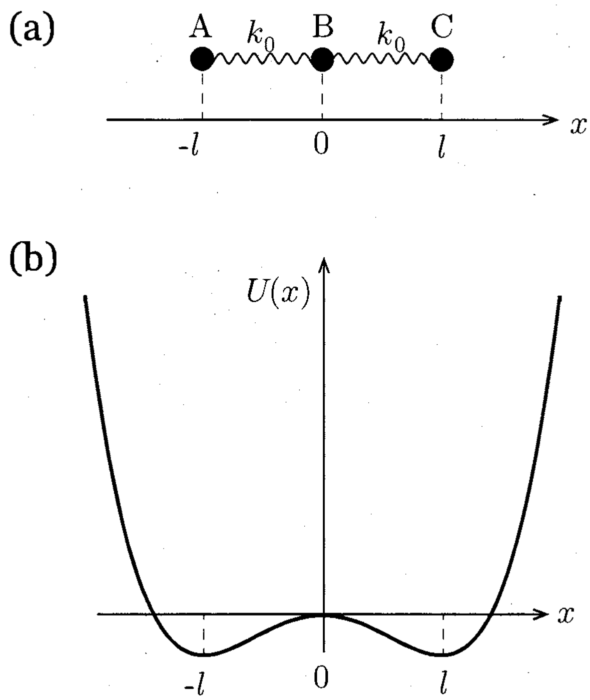

Figure 2

Problem 2

Two point charges of $\pm q$ ($q > 0$) separated by the distance d are placed in the vacuum as shown in Fig. 1. Consider the static electric field at a given point P. Let O be the midpoint of the two point charges, r be the distance from O to P, r_+ be the distance from the positive charge to P, r_- be the distance from the negative charge to P. Also, $\boldsymbol{d}$ is defined as the vector from the negative charge to the positive charge ($|\boldsymbol{d}| = d$), $\boldsymbol{p} = q\boldsymbol{d}$ is the electric dipole moment of the two point charges ($|\boldsymbol{p}| = p$), and θ is the angle between $\boldsymbol{p}$ and $\overrightarrow{\mathrm{OP}}$. The vacuum permittivity is ε_0.

[1] Express the electrostatic potential at P, $\phi(\mathrm{P})$, in terms of ε_0, q, r_+, and r_-.

[2] Express the approximate formula of the electrostatic potential $\phi(\mathrm{P})$ valid in the regime $r \gg d$ in terms of ε_0, d, q, r, and θ. Neglect the second- and higher-order terms of d/r.

[3] In the regime $r \gg d$, the electric field at P can be expressed by the following equation:

$$\boldsymbol{E}(r,\theta) = \frac{p}{4\pi\varepsilon_0 r^3}(2\cos\theta\ \boldsymbol{e}_r + \sin\theta\ \boldsymbol{e}_\theta). \tag{1}$$

Show how Equation (1) is derived from the result of [2]. Here $\boldsymbol{e}_r$ is the unit vector parallel to $\overrightarrow{\mathrm{OP}}$, and $\boldsymbol{e}_\theta$ is the unit vector perpendicular to $\boldsymbol{e}_r$ and within the plane spanned by $\boldsymbol{d}$ and $\boldsymbol{e}_r$, as shown in Fig. 1.

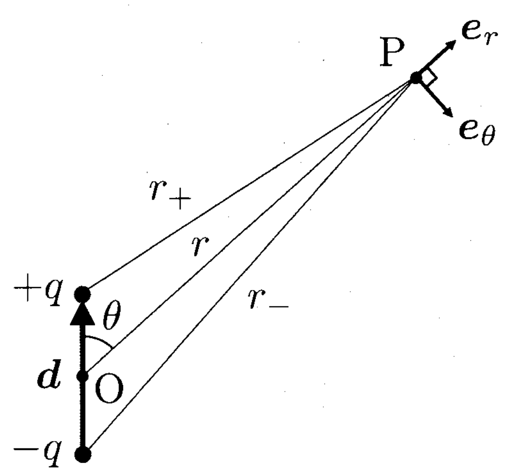

Figure 1

In the following, assume that the spatial extent of each electric dipole is negligible and that the electric field produced by each electric dipole is expressed by Equation (1). Below we abbreviate the term "electric dipole" as "dipole" for the sake of simplicity.

Two dipoles 1 and 2 are placed in the vacuum with the distance l apart, as shown in Fig. 2. Here the positions of the dipoles are fixed and their dipole moments $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ are allowed to rotate within the plane of the paper. We define θ_1 (θ_2) as the angle between $\boldsymbol{p}_1$ ($\boldsymbol{p}_2$) and the straight line passing through the dipoles 1 and 2.

[4] Suppose that the orientation of $\boldsymbol{p}_1$ is fixed and only $\boldsymbol{p}_2$ is allowed to rotate within the plane of the paper in Fig. 2. Starting from the configurations (a) $(\theta_1, \theta_2) = (\pi/2, 0)$ and (b) $(\theta_1, \theta_2) = (\pi/2, -\pi/2)$, what is the initial response of $\boldsymbol{p}_2$ to the electric field produced by the dipole 1? For each configuration, choose the correct answer from {clockwise rotation, counter-clockwise rotation, no rotation}.

[5] Express $U = -\boldsymbol{p}_2 \cdot \boldsymbol{E}_1$, which is the energy of the dipole 2 under the electric field $\boldsymbol{E}_1$, in terms of ε_0, p_1, p_2, l, θ_1 and θ_2. Here $\boldsymbol{E}_1$ is the electric field produced by the dipole 1, and p_1 (p_2) is the magnitude of $\boldsymbol{p}_1$ ($\boldsymbol{p}_2$).

[6] Suppose that the two dipoles in Fig. 2 are now free to rotate within the plane of the paper. The energy U derived in [5] is the potential energy of the system. Find the most stable configuration(s) (θ_1, θ_2) and express the corresponding U in terms of ε_0, p_1, p_2, and l. Also, describe how you reached these conclusions.

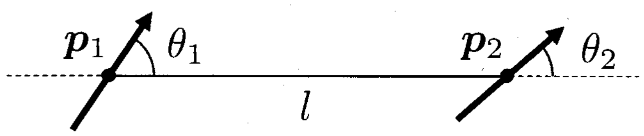

Figure 2

A dipole with the moment $\boldsymbol{p}$ and its magnitude p is placed in the vacuum at the distance h from the surface ($z = 0$) of an electrically grounded conductor which is infinitely extended along x and y directions, as shown in Fig. 3.

[7] Express the potential energy U' in terms of ε_0, p, h, and α, where α is the angle between $\boldsymbol{p}$ and the line normal to the conductor's surface. Set $U' = 0$ when h is infinitely large.

[8] Suppose that the position of the dipole is fixed at the distance h from the conductor's surface, and that its dipole moment is free to rotate within the plane of the paper. Find value(s) of α for the most stable orientation(s) of $\boldsymbol{p}$.

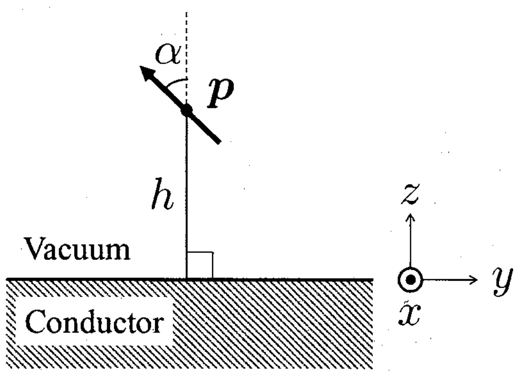

Figure 3

Department of Applied Physics
Entrance Examination Booklet

Physics II

(Answer 3 Problems among the 4 Problems in this Booklet)

August 30 (Tuesday) 13:00 – 16:00, 2016

REMARKS

1. Do not open this booklet before the start is announced.
2. Inform the staff when you find misprints in the booklet.
3. Choose three problems among the four problems in this booklet, and answer the three problems.
4. Use one answer sheet for each problem (three answer sheets are given). You may use the back side of each answer sheet if necessary.
5. Write down the number of the problem which you answer in the given space at the top of the corresponding answer sheet.
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Examinee number	No.

Write down your examinee number above

Problem 1

Consider a particle of mass m undergoing an one-dimensional motion under the potential

$$V(x) = \begin{cases} +\infty & (x < -\frac{a}{2}) \\ 0 & (-\frac{a}{2} \leq x \leq \frac{a}{2}) \\ +\infty & (x > \frac{a}{2}) \end{cases},$$

where a is a positive constant. Answer the following questions, using $\hbar = h/2\pi$, where h is the Planck constant.

[1] Consider the orbital states of a single particle. Find all of the energy eigenvalues and the corresponding normalized eigenfunctions. Here, ignore the spin degrees of freedom.

In the following, consider only the ground state and the first excited state obtained in [1], and write the corresponding wave functions as $g(x)$ and $e(x)$, respectively.

[2] Consider two non-interacting identical particles. Let $\Psi(x_1, x_2)$ be an orbital wave function of the two particles, where x_1 and x_2 are the coordinates of particle 1 and particle 2, respectively. Any wave function has either a symmetric form with $\Psi(x_1, x_2) = +\Psi(x_2, x_1)$ or an anti-symmetric form with $\Psi(x_1, x_2) = -\Psi(x_2, x_1)$ with respect to the permutation of the particles. Find all the four orbital wave functions for these two particles that are composed of $g(x)$ and/or $e(x)$. Ignore the spin degrees of freedom.

Next consider two identical Fermi particles with spin $s = \frac{1}{2}$. Let $\hat{\boldsymbol{s}}_1 = (\hat{s}_{1x}, \hat{s}_{1y}, \hat{s}_{1z})$ and $\hat{\boldsymbol{s}}_2 = (\hat{s}_{2x}, \hat{s}_{2y}, \hat{s}_{2z})$ be the spin operators of particle 1 and particle 2, respectively. Let $|\uparrow\rangle_1$ and $|\uparrow\rangle_2$ be the respective eigenstates of $\hat{s}_{1z}$ and $\hat{s}_{2z}$ with a positive eigenvalue, and $|\downarrow\rangle_1$ and $|\downarrow\rangle_2$ be the respective eigenstates with a negative eigenvalue. Let $\hat{\boldsymbol{S}} = \hat{\boldsymbol{s}}_1 + \hat{\boldsymbol{s}}_2 = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$ be the total spin operator of the two particles. Here, ignore the inter-particle interaction except for [6] and suppose that no magnetic field is applied.

[3] Consider only the spin degrees of freedom of the two particles. Find all the simultaneous eigenstates of $\hat{\boldsymbol{S}}^2$ and $\hat{S}_z$ along with their total spin S.

[4] Find six independent energy eigenstates, given by the products of the orbital states obtained in [2] and the spin states obtained in [3], as well as their corresponding eigenvalues.

[5] Evaluate the expectation value of the square distance $\langle (x_1 - x_2)^2 \rangle$ between the particles for each of the two-particle states obtained in [4]. For the sake of simplicity, the following integrals can be replaced by A, B, and C,

$$A = \int_{-\infty}^{\infty} x^2 g(x)^2 dx, \quad B = \int_{-\infty}^{\infty} x^2 e(x)^2 dx, \quad C = \int_{-\infty}^{\infty} x g(x) e(x) dx.$$

[6] Add the perturbation potential $V_0 x_1 x_2$ to the Hamiltonian. Evaluate the eigenvalue corresponding to each state, obtained in [4], up to the first order in V_0. You may use A, B, and C defined in [5].

Problem 2

Consider a chain-like flexible zigzag molecule on a line in thermal equilibrium at temperature T. For simplicity, the molecule is assumed to be made of N rods each of length a and linked together as schematically shown in Fig. 1. The angle between two adjacent rods is either 0 or π and each rod takes either right-hand or left-hand direction. The shape of this molecule is uniquely defined by aligning the N rods in a specific way from the origin to the end. For example, the shape of the molecule in Fig. 1 is described by (right, right, left, left, left, right, $\cdots$). The x axis is assumed to be along this molecule from the left to the right. The origin of the molecule is fixed at $x = 0$, and the coordinate of the end point of this molecule is denoted by X. Answer the following questions assuming that N is an even integer. The Boltzmann constant is denoted by k_B. Use the approximation $\log(M!) \cong M \log M - M$ (log denotes the natural logarithm) for an integer M ($\gg 1$).

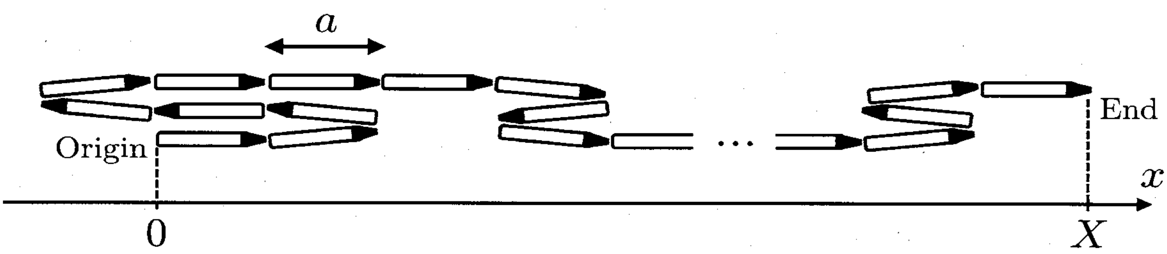

Figure 1

First, assume that the energy of the molecule does not depend on the directions of the rods.

[1] Express the coordinate of the end point X using the total number of right- and left-oriented rods, n^+ and n^-, respectively, and a. Express the probability $P(n)$ of $X = na$ using N and n, where n is an even integer satisfying $-N \leq n \leq N$. Derive the entropy S of the molecule using N, n and k_B, assuming that $N \pm n \gg 1$.

[2] Consider to fix the end point at $X(> 0)$. Assume that $N \pm X/a \gg 1$. Force τ necessary to fix the end point is expressed as

$$\tau = \left(\frac{\partial F}{\partial X}\right)_T,$$

where F denotes the Helmholtz free energy. Obtain τ. Furthermore, derive the relation between τ and X for $X \ll Na$.

Next, assume that an external field along the x axis is applied. Under this field, each rod attains an energy depending on its direction, i.e., $-\kappa$ for right-hand and $+\kappa$ for left-hand, where κ is a constant.

[3] Write down the partition function, and derive the expectation value of the energy, $\langle E \rangle$, and the expectation value of the coordinate of the end point, $\langle X \rangle$, as a function of T. Derive relevant expressions for $\langle E \rangle$ and $\langle X \rangle$ for $|\kappa| \ll k_\mathrm{B}T$.

[4] Derive the variance of the energy, $\langle E^2 \rangle - \langle E \rangle^2$, and the variance of the coordinate of the end point, $\langle X^2 \rangle - \langle X \rangle^2$, as a function of T.

Problem 3

As linearly polarized light passes through a uniform and isotropic medium under a static magnetic flux density $\boldsymbol{B}_{\rm ex}$, the oscillation plane of the electric field vector $\boldsymbol{E}$ rotates by an angle. This is called Faraday effect, and the rotation angle can be associated with the matrix elements of complex dielectric tensor $\tilde{\varepsilon}$. Hereafter, the permittivity and the permeability of vacuum are denoted by ε_0 and μ_0, respectively. The vector transpose is written as $(v_1, v_2, v_3)^{\rm T} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$.

First, we shall derive the complex dielectric tensor by considering the motion of bound electrons in the medium. Here, the electron mass is m and the electron charge is $-e$ (< 0). Under the spatially uniform complex electric field oscillating in time $\boldsymbol{E}_{\rm ex} = (E_x, E_y, 0)^{\rm T} \exp(-i\omega t)$ with $\omega > 0$, an electron is forced to vibrate with the complex displacement vector $\boldsymbol{u} = (u_x, u_y, 0)^{\rm T} \exp(-i\omega t)$. In addition to the force exerted by the electric field $\boldsymbol{E}_{\rm ex}$, the Lorentz force originating from the static magnetic flux density $\boldsymbol{B}_{\rm ex} = (0, 0, B)^{\rm T}$ $(B > 0)$ and the restoring force $-m\omega_0^2 \boldsymbol{u}$ $(\omega_0 > 0)$ are applied to the moving electron. Answer the questions [1] and [2], below.

[1] Write down the equation of motion of the electron and find u_x and u_y.

[2] The vector corresponding to the electric polarization density $\boldsymbol{P}$ of the medium is given by $\boldsymbol{P} = \varepsilon_0(\tilde{\varepsilon}\boldsymbol{E}_{\rm ex} - \boldsymbol{E}_{\rm ex}) = -ne\boldsymbol{u}$, where n is the electron density. The complex dielectric tensor $\tilde{\varepsilon}$ of the medium under a magnetic flux density $\boldsymbol{B}_{\rm ex}$ along the $+z$ direction has the following form,

$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & i\gamma & 0 \\ -i\gamma & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}. \tag{1}$$

Express ε_{xx} and γ as functions of ω and B. Then, show that there hold the relations $\gamma > 0$ and $\varepsilon_{xx} > 1$, when assuming $\omega_0 \gg \omega > 0$ and $\omega_0 \gg eB/m$.

Next, consider a plane wave propagating in the medium under the magnetic flux density $\boldsymbol{B}_{\rm ex}$ along the $+z$ direction, while keeping the polarization state intact. The electric field vector $\boldsymbol{E}$ and the magnetic field vector $\boldsymbol{H}$ of this plane wave is expressed as follows

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}_0 \exp\{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)\}, \quad \boldsymbol{H}(\boldsymbol{r}, t) = \boldsymbol{H}_0 \exp\{i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)\}. \tag{2}$$

Here, $\boldsymbol{k}$ is the wave vector, and $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$ are constant complex vectors independent of position $\boldsymbol{r} = (x, y, z)^{\rm T}$ and time t. This plane wave satisfies the following equations,

$$\boldsymbol{k} \times \boldsymbol{E}_0 = \mu_0 \omega \boldsymbol{H}_0, \quad \boldsymbol{k} \times \boldsymbol{H}_0 = -\tilde{\varepsilon}\varepsilon_0 \omega \boldsymbol{E}_0. \tag{3}$$

Assume that $\omega > 0$ and that $\tilde{\varepsilon}$ has the same form as that in Equation (1) with $0 < \gamma < \varepsilon_{xx}$.

[3] By eliminating $\boldsymbol{H}_0$ from Equations (3), derive the following equation,

$$(\boldsymbol{E}_0 \cdot \boldsymbol{k})\boldsymbol{k} - |\boldsymbol{k}|^2 \boldsymbol{E}_0 + \left(\frac{\omega}{c}\right)^2 \tilde{\varepsilon}\boldsymbol{E}_0 = \boldsymbol{0}, \tag{4}$$

where c is the speed of light in vacuum.

Further assume that the plane wave propagates along the $+z$ direction in the medium with a wave vector $\boldsymbol{k} = (0, 0, k_z)^{\mathrm{T}}$.

[4] Equation (4) has a solution for $\boldsymbol{E}_0$ with a nonvanishing x component only when $k_z = k_+$ and $k_z = k_-$ $(0 < k_- < k_+)$. Express $k_\pm$ using ε_{xx}, γ, ω, and c, and find the corresponding solutions $\boldsymbol{E}_0 = \boldsymbol{E}_{0\pm}$ normalized as $|\boldsymbol{E}_{0\pm}| = E$.

[5] Define $\boldsymbol{E}_\pm(z,t) = \boldsymbol{E}_{0\pm}\exp(ik_\pm z - i\omega t)$ using $\boldsymbol{E}_{0\pm}$ obtained in [4]. Sketch the time dependence of the real part of the x and y components of $\boldsymbol{E}_+(0,t)$.

As shown in Fig. 1, a plane wave propagating along $+z$ direction is incident on a medium with the thickness of l occupying the region $0 \le z \le l$. The plane wave is linearly polarized and its electric field vector has the following form at $z = 0$,

$$\boldsymbol{E}(\boldsymbol{r},t)\Big|_{z=0} = (E, 0, 0)^{\mathrm{T}} \exp(-i\omega t). \tag{5}$$

[6] Express $\boldsymbol{E}(\boldsymbol{r},t)$ that satisfies the boundary condition of Equation (5) as a linear combination of $\boldsymbol{E}_+(z,t)$ and $\boldsymbol{E}_-(z,t)$ defined in [5]. The electric field vector of the plane wave at $z = l$ can be expressed as

$$\boldsymbol{E}(\boldsymbol{r},t)\Big|_{z=l} = \boldsymbol{F}\exp\left(i\frac{k_+ + k_-}{2}l\right)\exp(-i\omega t). \tag{6}$$

Derive the vector $\boldsymbol{F}$. Considering θ as the rotation angle of the polarization plane of the plane wave passing through the medium, derive an expression for θ by k_+, k_- and l. Note that reflection at the surfaces can be neglected.

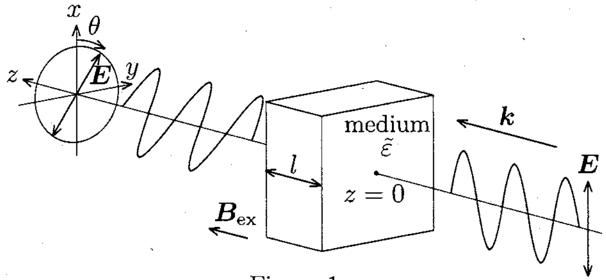

Figure 1

[7] As shown in Fig. 2, after being reflected by a mirror perpendicular to the z axis, the plane wave described in [6] with the rotation angle θ returns to the position $z = 0$. Find the rotation angle of the polarization plane relative to the x axis at this point.

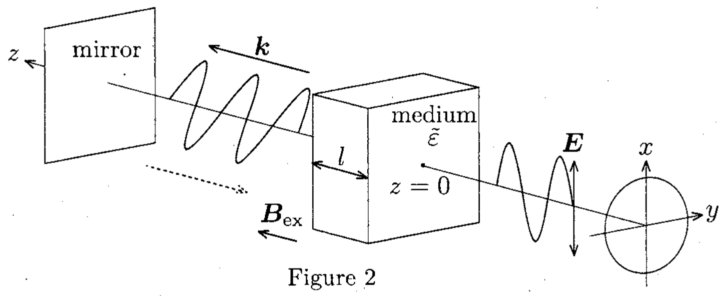

Figure 2

Problem 4

Consider an electron scattering experiment shown in Fig. 1. An electron beam with a wave vector $\boldsymbol{k}$ (wavelength: $\lambda = 2\pi/|\boldsymbol{k}|$) is incident on a sample, and the scattered beam with a wave vector $\boldsymbol{k}'$ is observed by a planar detector. The incident electron beam is regarded as a plane wave, and only elastic scattering is taken into account. Ignore multiple scattering processes. Answer the following questions.

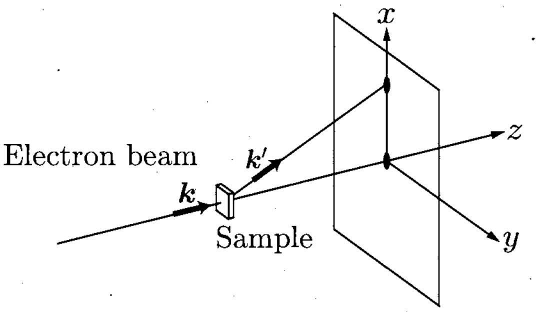

Figure 1

[1] Find the acceleration voltage to produce an electron beam with the wavelength $\lambda = 0.01$ nm. Use the following physical constants and neglect any relativistic correction.
Planck constant: $h = 6.6 \times 10^{-34}$ Js, electron charge: $-e = -1.6 \times 10^{-19}$ C
electron mass: $m = 9.1 \times 10^{-31}$ kg

Consider a sample composed of an arbitrary number of atoms. With $\boldsymbol{r}_j$ being the coordinate of the j-th atom, the amplitude $A(\boldsymbol{K})$ of the electron wave scattered by the vector $\boldsymbol{K}(= \boldsymbol{k}' - \boldsymbol{k})$ is expressed as

$$A(\boldsymbol{K}) = \sum_j f_j \exp(-i\boldsymbol{K} \cdot \boldsymbol{r}_j),$$

where f_j is the atomic scattering factor. In what follows, ignore the $\boldsymbol{K}$ dependence of the atomic scattering factor.

[2] Consider a sample composed of identical atoms with $f_j = f$. The atoms are arranged such that they form a square lattice with the lattice spacing a as shown in Fig. 2. The number of atoms along x (y) direction is defined as N_x (N_y) without any stacking along z. The position of the left-bottom atom is set to be the origin of the coordinates. Show that $A(\boldsymbol{K})$ corresponding to the scattering vector $\boldsymbol{K} = (K_x, K_y, K_z)$ can be described by a function L as

$$A(\boldsymbol{K}, N_x, N_y) = fL(K_x, N_x)L(K_y, N_y).$$

The scattering intensity is proportional to $|A(\boldsymbol{K}, N_x, N_y)|^2$. Draw the schematic graphs of $|L(K_x, N_x)|^2$ for $N_x = 1, 2,$ and 3, respectively, in the range of $0 \leq K_x \leq 2\pi/a$.

[3] Find the primitive reciprocal lattice vectors for the two-dimensional single-layer crystal with the rhombic unitcell containing one atom as shown in Fig. 3. For an electron beam vertically incident on the crystal, obtain the three smallest diffraction angles (the angle between $\boldsymbol{k}$ and

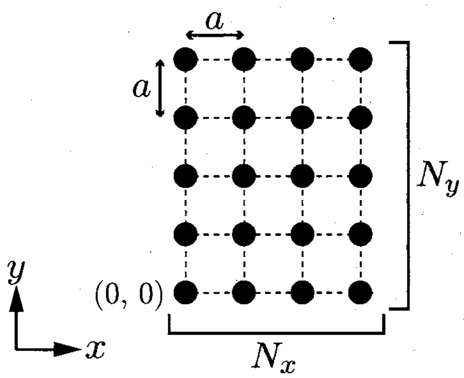

Figure 2

$\boldsymbol{k}'$) in the ascending order ($0 < \theta_1 < \theta_2 < \theta_3$), using the lattice constant a of the crystal and the wavelength λ of the electron beam. Assume that the number of atoms in the two-dimensional crystal is sufficiently large along both directions $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$. Additionally, draw the electron diffraction pattern on the planar detector corresponding to θ_1, θ_2, and θ_3 in the case of small diffraction angles.

Next, consider electron beam diffraction by a single crystalline sample whose unitcell contains multiple atoms. Assuming that the α-th atom in the n-th unitcell possesses the coodinate $\boldsymbol{R}_n + \boldsymbol{r}_\alpha$ and the atomic scattering factor f_α, $A(\boldsymbol{K})$ is expressed as

$$A(\boldsymbol{K}) = \sum_n \sum_\alpha f_\alpha \exp\left[-i\boldsymbol{K} \cdot (\boldsymbol{R}_n + \boldsymbol{r}_\alpha)\right].$$

[4] Consider the electron diffraction from a honeycomb structure made of regular hexagons containing two different atoms in its unitcell as shown in Fig. 4. Suppose that the ratio of the atomic scattering factors between atom A and atom B is $f_A/f_B = 1/2$. Classify the diffraction peaks obtained in [3] in terms of their intensities and find the corresponding relative intensities.

[5] Consider the electron diffraction for a layered crystalline sample where the atomic layers shown in Fig. 3 are stacked vertically with an equal spacing. The arrangement and positions of atoms are kept the same for all layers. When the number of stacking layers is sufficiently large, the diffraction peaks obtained in [3] are not observed with an electron beam vertically incident on the stacking layers. These diffraction peaks are observed when the stacking layers of the sample are slightly tilted against the incident electron beam. Describe the reason.

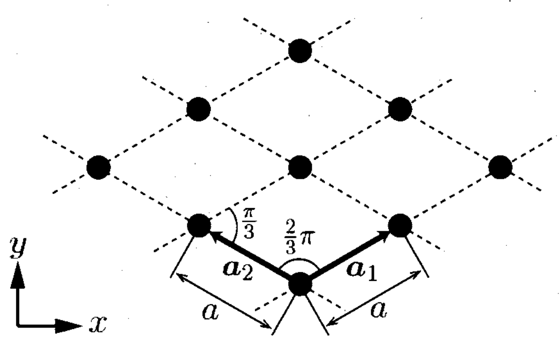

Figure 3

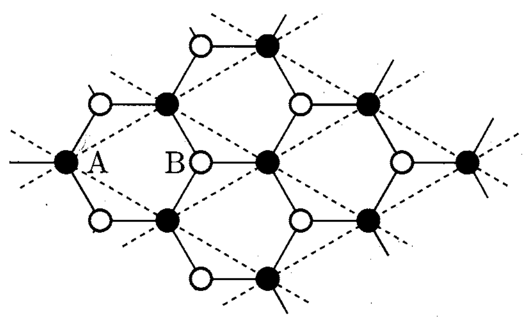

Figure 4